Student name: Wednesday 7 May 2014

Comp no#: Math 205 2nd Periodic Exam

Question (I): 7 Marks State whether the following is (True) or (False) and correct the false one

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1-( ) If f has a continuous partial derivatives of all orders on R^3, then $div\ (curl\ \nabla f\ ) = \overrightarrow{0}$

2-( ) The fundamental theorem of line integrals can be applied by :

$$\int_C \nabla f \bullet d\ \overrightarrow{r} = f(\overrightarrow{r}(b)) - f(\overrightarrow{r}(a)) \text{ for } \ a \leq t \leq b.$$

3-( ) $\oint_C \overrightarrow{F} \cdot \overrightarrow{n}\ ds = \iint_D curl\ \overrightarrow{F}\ dA$

4-( ) The $Surface\ area$ is given by : $A(S) = \iiint_S \sqrt{1 + (\frac{\partial f}{\partial x})^2 + (\frac{\partial f}{\partial y})^2 + (\frac{\partial f}{\partial z})^2} dv$

5-( ) The $flux$ of $\overrightarrow{F}$ across S is the Surface integral of $\overrightarrow{F}$ over the surface S and calculated by :

$$\iint_S \overrightarrow{F} \bullet d\overrightarrow{S} = \iint_S \overrightarrow{F} \bullet \overrightarrow{k}\ d\overrightarrow{S} = \iint_D (-P\frac{\partial g}{\partial x} - Q\frac{\partial g}{\partial y} + R)\ dA$$

6-( ) By Stoke's Theorem the line integral $\int_C \overrightarrow{F} \bullet d\overrightarrow{r} = \iint_S div\ \overrightarrow{F} \bullet d\overrightarrow{S}$

7-( ) If S is a shere and $\overrightarrow{F}$ is a constant vector field, then $\iint_S \overrightarrow{F} \bullet d\overrightarrow{S} = 0$

Solve the following Questions :
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(1) 4 Marks

Evluate the following line integral: $\int_C \overrightarrow{F} \bullet d\overrightarrow{r}$ where $\overrightarrow{F}(x, y) = \langle\, xy, x^2 \rangle$ and C is given by $\overrightarrow{r}(t) = \langle\, \sin t, (1+t)\,\rangle \quad 0 \le t \le \pi$

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(2) 6 Marks

(i) Show that $\overrightarrow{F}$ is a conservative vector field

(ii) Find the function f such that $\overrightarrow{F} = \nabla f$

(iii) Evaluate $\int_C \overrightarrow{F} \bullet d\overrightarrow{r}$ where $\overrightarrow{F}(x, y) = \langle 4x^3y^2 - 2xy^3 \,,\, 2x^4y - 3x^2y^2 + 4y^3 \rangle$

and $C : \overrightarrow{r}(t) = \langle\, t + \sin \pi t \,,\, 2t + \cos \pi t\,\rangle \,,\, 0 \le t \le 1$

(3) 4 Marks Use Green's Theorem to evaluate $\int_C \sqrt{1+x^3}\,dx + 2xy\,dy$ where C is the triangle

with vertices(0,0) , (1,0) and (1,3)

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(4) 4 Marks Find the *flux* of $\overrightarrow{F}\ (x, y, z) = \langle xy, yz, xz\rangle$,S is the part of the paraboloid $z = 4 - x^2 - y^2$ that lies above the square $0 \leq x \leq 1$, $0 \leq y \leq 1$ and has upward orientation.

Good Luck

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